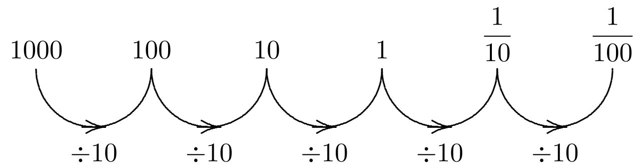
8. DECIMALS

extending place values to the right of the ones place;

the decimal point

Consider again the place values in our base ten number system. If we move from left to right, notice that the place value is successively divided by ten:



- $\frac{1000}{10} = 100$; the thousands place is followed by the hundreds place
- $\frac{100}{10} = 10$; the hundreds place is followed by the tens place
- $\frac{10}{10} = 1$; the tens place is followed by the ones place

tenths place; hundredths place; and so on

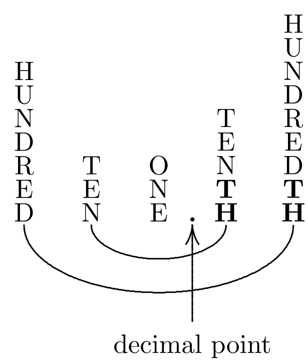
This pattern continues, after first putting a *decimal point* to the right of the ones place:

- one divided by ten is one-tenth; the first place to the right of the decimal point is the *tenths* place;
- one-tenth divided by ten is one-hundredth; the second place to the right of the decimal point is the *hundredths* place; and so on.

hundred versus hundredth

Be certain to notice the difference between *hundred* and *hundredth*. *Hundred* is to the left of the decimal point, and *hundredth* is to the right of the decimal point.

The place values are 'mirrored' about the ones place, adding '*th*' to the right of the decimal point.



decimals

A base ten number that uses a decimal point is called a *decimal*. Thus, 2.5 and 0.0003 are called decimals, but 3 is not called a decimal. If there are no digits to the left of the decimal point, then it is good practice to put a zero in the ones place: that is, write 0.02 , not .02 .

using exponent notation for the place values to the right of the decimal point

The first place to the right of the decimal point has place value $\frac{1}{10}$.
The second place to the right of the decimal point has place value $\frac{1}{10^2}$.
In general, the n^{th} place to the right of the decimal point has place value $\frac{1}{10^n}$.

$$\begin{array}{lccccc}
\text{place values} \longrightarrow & 1 & & \frac{1}{10} & \frac{1}{10^2} & \frac{1}{10^3} \\
 & \underline{\text{x}} & . & \underline{\text{x}} & \underline{\text{x}} & \underline{\text{x}} \\
 & & \uparrow & & & \\
 & & \text{decimal point} & & &
\end{array}$$

When exponent notation is studied in more detail in a future section, you'll see that 1 can be written as 10^0 ; $\frac{1}{10}$ can be written as 10^{-1} ; $\frac{1}{10^2}$ can be written as 10^{-2} , etc. Thus, there's a beautiful pattern in the entire place value scheme:

$$\begin{array}{lcccccccc}
\text{place values} \longrightarrow & 10^3 & 10^2 & 10^1 & 10^0 & & 10^{-1} & 10^{-2} & 10^{-3} \\
 & \underline{\text{x}} & \underline{\text{x}} & \underline{\text{x}} & \underline{\text{x}} & . & \underline{\text{x}} & \underline{\text{x}} & \underline{\text{x}} \\
 & & & & & \uparrow & & & \\
 & & & & & \text{decimal point} & & &
\end{array}$$

EXERCISES

1. What is the place value of the fifth place to the right of the decimal point?
2. What is the place value of the hundredth place to the right of the decimal point?
3. Suppose you're at a certain position in a decimal where the place value is $\frac{1}{10^9}$. What is the place value of one position to the right? What is the place value of one position to the left?

reading decimals aloud

To read decimals aloud, start by using the prior rules for reading the part to the left of the decimal point. Read the decimal point as 'and'. Only the right-most place value is used for reading the part to the right of the decimal point, as illustrated in the following examples:

- read 2.03 as *two and three hundredths* .
- read 23.457 as *twenty-three and four hundred fifty-seven thousandths* .
- read 0.000042 as *forty-two millionths* .

Notice that the word 'and' should ONLY be used for the decimal point. Resist the temptation to insert the word 'and' anywhere else!

EXERCISES

4. Read each of the following decimals aloud:
   a. 3.57
   b. 247.0921
   c. 0.000000005
   d. 2000.03

an alternate way to read decimals

Reading a decimal like 972.28936 following the rules above gets a bit tedious. Thus, it is often read as *nine hundred seventy-two point two, eight, nine, three, six*. That is, say 'point' to represent the decimal point, and then just read each digit, separately, that follows the decimal point.

EXERCISES

5. Read each of the following decimals aloud in the 'alternate' way:
   a. 453.57129
   b. 247.09213
   c. 0.08714
   d. 2000.032

renaming decimals as fractions

The number 0.237 can be viewed as

$$2 \cdot \tfrac{1}{10} + 3 \cdot \tfrac{1}{100} + 7 \cdot \tfrac{1}{1000}$$

or can alternately be viewed as

$$237 \cdot \tfrac{1}{1000} = \tfrac{237}{1000}\,.$$

fractions $\frac{N}{D}$; N is the numerator; D is the denominator

Recall that in a fraction $\frac{N}{D}$, the top is called the *numerator* and the bottom is called the *denominator*. For example, in the fraction $\frac{23}{100}$, the numerator is 23 and the denominator is 100 .

going from a decimal to a fraction

Thus, to go from a a decimal to a fraction, you use the right-most place value to determine the correct denominator; the entire number (without the decimal point) becomes the numerator. In particular, the number of zeros in the denominator is the same as the number of places to the right of the decimal point. Here are some more examples:

$0.0013 = \frac{13}{10000}$ (four places to the right of the decimal point; four zeros in the denominator)

$23.107 = \frac{23107}{1000}$ (three places to the right of the decimal point; three zeros in the denominator)

$0.72 = \frac{72}{100}$ (two places to the right of the decimal point; two zeros in the denominator)

If you're rusty on fractions, don't worry—they will be reviewed in a future section. Also, don't worry about simplifying fractions at this point: you can leave 0.4 as $\frac{4}{10}$, instead of reducing it to $\frac{2}{5}$.

factors

In any multiplication problem, the numbers being multiplied are called the *factors*. For example, in the multiplication problem $23.1 \cdot 10$, the factors are 23.1 and 10 .

multiplying by powers of ten

To multiply a decimal by powers of ten, you just move the decimal point one place to the right for each factor of ten. Here are some examples. The $\times$ symbol is used for multiplication in these problems, because the centered dot is too easily confused with the decimal point.

$23.19 \times 10 = 231.9$: move the decimal point one place to the right

$7.001 \times 10^3 = 7001$: move the decimal point three places to the right

$0.03 \times 10^4 = 300$: move the decimal point four places to the right, inserting zeros as needed *dividing by powers of ten*

To divide a decimal by powers of ten, you just move the decimal point one place to the left for each factor of ten. Here are some examples. Remember that division can be denoted using either the $\div$ symbol, or a horizontal fraction bar.

$23.1 \div 10 = 2.31$; move the decimal point one place to the left

$\frac{7.001}{1000} = 0.007001$; move the decimal point three places to the left, inserting zeros as needed

$37.2 \cdot \frac{1}{1000} = \frac{37.2}{1000} = 0.0372$: multiplying by $\frac{1}{1000}$ is the same as dividing by 1000 .

Make sure you understand why this works! For example, when 2.37 is divided by 10 , the 2 *ones* should turn into 2 *tenths*. Moving the decimal point one place to the left accomplishes this.

MR. DILL

You may want to come up with a memory device to help you remember:

- **M**ultiply by powers of ten, move the decimal point to the **R**ight;
- **D**ivide by powers of ten, move the decimal point to the **L**eft.

A key phrase like **MR. D**oo**L**ittle may help you remember until the process gets into long-term memory.

EXERCISES

6. Do the following calculations without a calculator:
   a. $4.372 \cdot 10^2$
   b. $0.037 \cdot 10{,}000$
   c. $243 \div 100$
   d. $\frac{243.5}{1000}$
   e. $\frac{0.03}{10^2}$
   f. $347.2 \div 10^4$
   g. $27.9 \cdot \frac{1}{1000}$
   h. $3 \cdot \frac{1}{10^2}$

percents

One use for decimals is in working with *percents*, which are commonplace in everyday life:

- the dress was on sale for 40% off the original price;
- housing costs rose 5% last year;
- there was a 150% increase in telephone activity after the newspaper advertisement.

Whenever computations need to be done with percents, the percents are first renamed as decimals. This section concludes with an introduction to percents; they will be studied in more detail in a future section.

'per cent' means 'per one hundred'

There are 100 **cent**s in a dollar. A **cent**ury is 100 years. The word 'percent' means 'per one hundred'.

The symbol % is used for percent. Whenever you see the symbol '%', you can trade it in for a factor of $\frac{1}{100}$. Whenever you see a factor of $\frac{1}{100}$, it can be traded in for a % symbol. This simple idea is the key to success with percents:

$$\boxed{\% = \frac{1}{100}}$$

Indeed, the symbol % even looks like the fraction $\frac{1}{100}$; it has the two zeros and the division bar!

changing percents to decimals

Here are examples of changing percents to decimals. The % symbol is replaced with a factor of $\frac{1}{100}$:

$3.5\% = 3.5 \cdot \frac{1}{100} = 0.035$

$25\% = 25 \cdot \frac{1}{100} = 0.25$

$50\% = 50 \cdot \frac{1}{100} = 0.5$

$100\% = 100 \cdot \frac{1}{100} = 1$

$250\% = 250 \cdot \frac{1}{100} = 2.5$

As these examples illustrate, the percent symbol instructs division by 100 , which is accomplished by moving the decimal point two places to the left. Remember that if you don't see a decimal point, it gets inserted just to the right of the ones place.

Now, you can go from percents to decimals in one easy step:

$3\% = 0.03$

$2.37\% = 0.0237$

$0.01\% = 0.0001$

$5032\% = 50.32$

changing decimals to percents

To change from a decimal to a percent, move the decimal point two places to the right and insert the percent symbol:

$0.03 = 3\%$

$2.5 = 250\%$

$1.008 = 100.8\%$

Here's the idea that makes this work:

$$0.03 = 3 \cdot \frac{1}{100} = 3\%$$
$$2.5 = 2.50 = 250 \cdot \frac{1}{100} = 250\%$$

Notice that the $\frac{1}{100}$ gets traded in for the percent symbol.

P*u***D***d***L***e* **D***ip***P***e***R**

Memory devices are helpful until procedures get into long-term memory. You might want to remember these rules with the phrase **P**u**D**d**L**e **D**ip**P**e**R** :

- **P**ercent to **D**ecimal, move decimal point two places to the **L**;
- **D**ecimal to **P**ercent, move decimal point two places to the **R** .

(A 'puddle dipper' is a little kid who dips her toes in puddles!)

EXERCISES

7. Convert each percent to a decimal:
   a. 5%
   b. 75%
   c. 0.3%
   d. 340%
8. Convert each decimal to a percent:
   a. 0.07
   b. 2.1
   c. 0.358
   d. 0.8

EXERCISES *web practice*	Go to my homepage **http://onemathematicalcat.org** and navigate to my Algebra I course, which has about 170 sequenced lessons. It can be used as a complete year-long high school course, or one semester in college. You're currently looking at the pdf version—you'll see that the HTML version has unlimited, randomly-generated, online and offline practice in every section. It's all totally free. Enjoy!

SOLUTIONS TO EXERCISES: DECIMALS

1. $\frac{1}{10^5}$; the hundred-thousandths place
2. $\frac{1}{10^{100}}$
3. one to the right of $\frac{1}{10^9}$ is $\frac{1}{10^{10}}$; one to the left of $\frac{1}{10^9}$ is $\frac{1}{10^8}$.
4. a. three and fifty-seven hundredths

b. two hundred forty-seven and nine hundred twenty-one ten-thousandths

c. five billionths

d. two thousand and three hundredths

5. a. 453.57129 : four hundred fifty-three point five, seven, one, two, nine

b. 247.09213 : two hundred forty-seven point zero, nine, two, one, three

c. 0.08714 : zero point zero, eight, seven, one, four

d. 2000.032 : two thousand point zero, three, two

6. a. $4.372 \cdot 10^2 = 437.2$

b. $0.037 \cdot 10{,}000 = 370$

c. $243 \div 100 = 2.43$

d. $\frac{243.5}{1000} = 0.2435$ Remember to put the zero before the decimal point!

e. $\frac{0.03}{10^2} = 0.0003$

f. $347.2 \div 10^4 = 0.03472$

g. $27.9 \cdot \frac{1}{1000} = 0.0279$

h. $3 \cdot \frac{1}{10^2} = 0.03$

7. a. $5\% = 0.05$

b. $75\% = 0.75$

c. $0.3\% = 0.003$

d. $340\% = 3.4$

8. a. $0.07 = 7\%$

b. $2.1 = 210\%$

c. $0.358 = 35.8\%$

d. $0.8 = 80\%$