CHOOSING YOUR NUMBER!!

Instead of just *assigning* each girl a number, I wanted to let each of you choose *choose your own number* for the year. However, I really didn't want to deal with lots of 'adjustments' due to duplicate number choices. So, in preparing the 'choosing a number' activity, I asked myself the question:

(*) HOW LIKELY IS IT THAT TWO (or more) GIRLS WOULD CHOOSE THE SAME NUMBER?

The purpose of this activity is to discuss how I went about answering this question, and to show you how the answer influenced the 'choosing a number' activity on the first day of class.

If you finish this activity before the end of class, please pass it in to me.

Otherwise, please complete it for homework and pass it in as you come into class tomorrow. Thanks!

This activity will count as a HOMEWORK grade.

There are $\boxed{\text{BOXED}}$ items throughout these sheets. Each $\boxed{\text{boxed}}$ item requires student input.

The first thing you'll notice is that the answer to question (*) depends on a few things:

- HOW MANY GIRLS ARE IN THE CLASS?

The greater the number of girls, the (circle one) MORE LESS likely it is that we'll get a duplicate.

- HOW MANY NUMBERS ARE YOU BEING ALLOWED TO CHOOSE FROM?

For example, if you're allowed to choose any number from 1 to 10, then you're choosing from 10 numbers.

If you're allowed to choose any number from 100 to 200, then you're choosing from 101 numbers.

(Think about this . . .

Suppose you have books on a shelf, numbered from 1 to 200.

You really only want the books from 100 to 200.

So what might you do? You'd REMOVE books 1 through 99.

How many are left?

You started with 200; you took away 99; leaving you with $200 - 99 = 101$.)

The more numbers that you're choosing from, the (circle one) MORE LESS likely it is that a duplicate number will be chosen.

Now, let's set things up mathematically (of course!):

Suppose there are m girls in the class, who are allowed to randomly choose from N different numbers.

In our class, $m =$ ______________.

Yesterday, I let you select any number from __________ to __________. Therefore, $N =$ __________.

Now we need to ask: How many different ways are there for m girls to select from N different numbers?

Often, it's helpful to look at a simple example. (This is an EXCELLENT strategy for problem-solving. Often, if you can understand what to do in a simple case, then you can figure out what to do in a more complicated case.)

Suppose there are only two girls (call them girl1 and girl2) choosing from three different numbers (say, 7, 8, and 9).

In this simple example, $m =$ __________ .

In this simple example, $N =$ __________ .

Imagine performing the following 'experiment':

Call girl1 out of the room. Ask her what number she chose. Write it down.

Call girl2 out of the room. Ask her what number she chose. Write it down.

Here are some of the possible outcomes of this 'experiment':

girl1 chose the number ...	girl2 chose the number ...
7	8
9	9
8	7

In the space provided below, please list ALL the possible ways that two girls could choose from the three numbers 7, 8, and 9. (That is, list all the possible outcomes of this 'experiment' . TRY TO LIST THEM IN AN ORGANIZED WAY.

How many possible outcomes are there? (Note: 'girl1 chooses 7 and girl2 chooses 8' is ONE outcome.)

Which of the outcomes correspond to our desired situation: *the girls choose different numbers*? (Please CIRCLE these outcomes in your list above.)

What are the WHOLE NUMBERS?

Fill in the blanks with whole numbers: Out of __________ possible ways that two girls might choose from three different numbers, __________ of these correspond to the girls choosing DIFFERENT numbers.

What is the likelihood (probability) that, if two girls get to select from three numbers, they'll end up with different numbers? Please express your answer as both a FRACTION and as a PERCENT. (For example, $\frac{1}{2} = 50\%$.) ROUND your percent answer to the tenths place.

What is the likelihood (probability) that, if two girls get to select from three numbers, they WON'T end up with different numbers? Please express your answer as both a FRACTION and as a PERCENT (rounded to the tenths place).

The general situation (with m girls randomly choosing from N numbers) uses all the same ideas. However, it definitely looks more complicated because of the variables involved. Read through the following discussion, but don't be too hard on yourself—you may not yet have the mathematical skills necessary to follow everything. (However, notice that there ARE a couple questions embedded in this material, so don't just skip over it!)

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How many different ways are there for m girls to select from N different numbers?

There are N choices for girl1 ;

N choices for girl2 ;

$\vdots$,

N choices for girl$m$. Together, there are

$$\overbrace{N \cdot N \cdot \ldots \cdot N}^{m \text{ factors}} = N^m$$

possible ways.

Look back at the simple example, with two girls choosing from the numbers 7, 8, and 9 . Calculate N^m . Does this agree with your earlier work?

Since there are LOTS of ways that duplicates could occur (think about this!), it's actually easier to count how many outcomes have NO DUPLICATES. Here's how.

Let girl1 choose from any of the N numbers. So, she has N possible choices.

Now, girl2 only gets to choose from $N - 1$ numbers. (She can't choose the number that girl1 chose.)

Next, girl3 gets to choose from $N - 2$ numbers. (Do you see a pattern forming here?)

Continue in this way ...

Finally, girl$m$ has $N - (m - 1)$ choices.

So, how many ways can m girls choose from N numbers, so that there WON'T be any duplicates?

$$N \cdot (N-1) \cdot (N-2) \cdot \ldots \cdot (N-(m-1)) \ .$$

Look back at the special case you did, with two girls choosing from the numbers 7, 8, and 9 . Calculate $N \cdot (N-1) \cdot (N-2) \cdot \ldots \cdot (N-(m-1))$. Does this agree with your earlier work?

(By the way, there's a simpler name for the number $N \cdot (N-1) \cdot (N-2) \cdot \ldots \cdot (N-(m-1))$, if you know about *factorial notation*.)

So, the likelihood (probability) that there won't be ANY duplicates is:

$$\frac{\text{number of outcomes with no duplicates}}{\text{total number of outcomes}} = \frac{N \cdot (N-1) \cdot (N-2) \cdot \ldots \cdot (N-(m-1))}{N^m} \ .$$

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Let's practice using this formula.

Suppose three girls are choosing from five different numbers. Thus, $m = 3$ and $N = 5$. The likelihood that there won't be ANY duplicates is:

$$\frac{5 \cdot 4 \cdot 3}{5^3} = 0.48 = 48\% \ .$$

(Be sure that you do this on your calculator!)

If we now let the same three girls choose from ten different numbers ($m = 3$ and $N = 10$), we get:

$$\frac{10 \cdot 9 \cdot 8}{10^3} = 0.72 = 72\% \ .$$

So now, there's a 72% chance that no duplicate(s) will occur.

Now, you try some:

Suppose that four girls are choosing from seven different numbers. What is the likelihood that there won't be ANY duplicates? Express your answer as a percent, rounded to the units place.

Suppose that thirteen girls (Mrs. Fisher's largest class this year) are choosing from 223 different numbers. (This was the actual situation in Mrs. Fisher's largest class on Monday.) What is the likelihood that there won't be ANY duplicates? Express your answer as a percent, rounded to the units place.

Realistically, when I give a number range like 101—324, each number is probably not *quite* equally likely to be chosen. Certain number(s) seem to be more 'appealing' than others. Is/are there any number(s) in this range that you think might actually be *more likely* to be chosen? WHY? (By the way—if the numbers aren't equally likely, then the mathematical analysis of the situation is more difficult!)