# SAMPLE PREREQUISITE PROBLEMS: PRECALCULUS

Problems 1–13 should be done WITHOUT A CALCULATOR.

1. For each expression given below, rename the expression as requested. If the requested name is not possible, so state. A few samples are done for you.

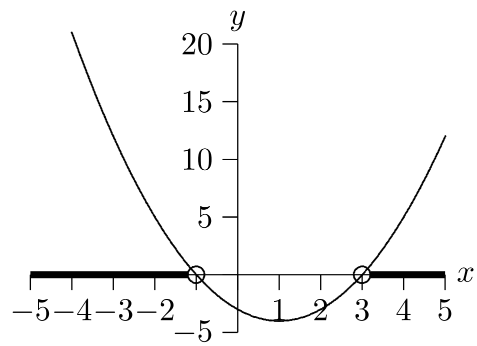
| | EXPRESSION | RENAME IN THIS FORM | ANSWER |
|---|---|---|---|
| | (sample) 12 | a sum of even integers | $2 + 10$ or $4 + 8$ etc. |
| | (sample) 12 | $2^x$ , where $x \in \{0, 1, 2, 3, \dots\}$ | not possible |
| a) | $\dfrac{1}{\sqrt{2}}$ | a fraction with no radical in the denominator | |
| b) | 23,070,000 | in scientific notation | |
| c) | $x^2 - y^2$ | as a product (i.e., factor) | |
| d) | $\dfrac{x^4 x^{-1}}{(x^2)^3 x}$ | $x^k$ | |
| e) | 300 ft/sec | $x$ mph (there are $5,280$ feet in one mile) | |
| f) | 7,036 | $x \cdot 10^2 + y \cdot 10^{-1}$ | |
| g) | $8^{-2/3}$ | as a simple fraction | |
| h) | $x^2 + 2x + 3$ | involving a perfect square, $(x + k)^2$ | |
| i) | $\lvert 2x + 3 \rvert$, for $x < -\frac{3}{2}$ | without absolute values | |
| j) | $2\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ | $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ | |
| k) | $2 + 3i - i^2 + (1 - i)(3 + 4i)$ , $(i = \sqrt{-1})$ | $a + bi$ | |
| l) | $\dfrac{x^2 - x - 1}{x - 3}$ | $Q(x) + \frac{R(x)}{D(x)}$ | |
| m) | $\log_7 5$ | involving the natural log | |
| n) | $\dfrac{4 \log 10^x}{3}$ | without logarithms | |
| o) | $\ln x^4 - \ln x^2 + \ln(x^2 + 1)$ | a single logarithm | |
| p) | $(x - 2y)^4$ | expanded form (Hint: use Pascal's triangle) | |
| q) | $(-\infty, -2] \cap (-4, 5]$ | as a single interval | |
| r) | $\{x \mid x \geq -2\}$ | using interval notation | |

2. Solve each equation/inequality/system. Get EXACT answers, not decimal approximations. Report each solution set using correct set notation. A sample is done for you.

(sample) $x^2 - 2x > 3$

Solution: Rewrite:
$$x^2 - 2x - 3 > 0$$
$$(x-3)(x+1) > 0$$

(graph $y = (x-3)(x+1)$; see where graph lies above $x$-axis and read off solution set)
$$x < -1 \quad \text{or} \quad x > 3$$
$$\text{Solution set: } (-\infty, -1) \cup (3, \infty)$$




(a) $3x(1-5x)(x^2-16) = 0$

(b) $\frac{1}{2}x - 7 = 3x + \frac{x}{5}$

(c) $|2x-3| > 5$

(d) $2 < |x| < 3$

(e) $1 - 2x \leq 3$ or $-3 \leq x < -2$

(f) $x^2 = x + 2$

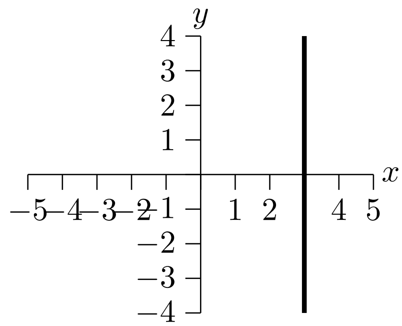
(g) $\sqrt{3x^2 + 5x - 3} = x$

(h) $y = x^2 + 1$ and $y = 2x + 4$

3. Graph each of the following equations/inequalities, where each sentence is viewed as a sentence in two variables. (That is, $x = 3$ should be viewed as $x + 0y = 3$.) A sample is done for you.

(sample) $x = 3$

Solution:




(a) $x > 3$

(b) $2y - 3 = 0$

(c) $x = 3$ and $y = 2$

(d) $x = 3$ or $y = 2$

(e) $y - 2x + 1 = 0$

(f) $y = -2\sqrt{x+3} + 1$

(g) $|x| = 2$

(h) $y \leq 2$

(i) $\frac{y-2}{3} = 2x - 1$

(j) $\frac{y-2}{3} \geq 2x - 1$

(k) $x^2 + 2x + y^2 - 6y - 15 = 0$

4. Write a list of transformations that takes the graph of $y = f(x)$ to the graph of $y = 5 - 3|f(x+1)|$. There may be more than one correct answer.

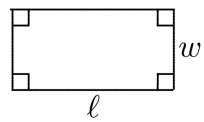
| EQUATION | TRANSFORMATION |
|---|---|
| $y = f(x)$ | (starting place) |
| | |
| | |
| | |
| | |
| | |
| | |

5. Starting with the equation $y = x^2 - 2x + 1$, apply the specified sequence of transformations.

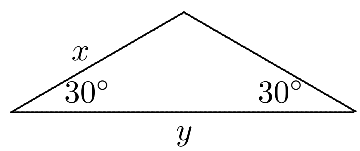
| EQUATION | TRANSFORMATION |
|---|---|
| $y = x^2 - 2x + 1$ | (starting place) |
| | up 1 |
| | left 3 |
| | reflect about the $x$-axis |
| | vertical scale by a factor of 2 |

6. Find the requested measurement(s) of each geometric figure.

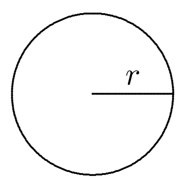
(a) PERIMETER and AREA:



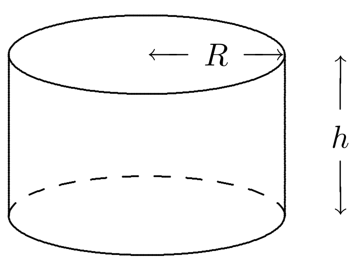

(b) PERIMETER and AREA:




(c) CIRCUMFERENCE and AREA:




(d) VOLUME:



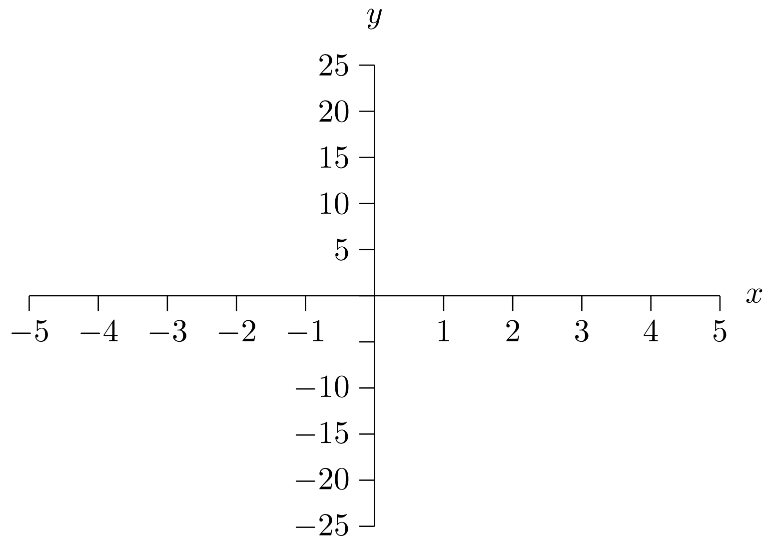

Which of the units below is a unit of length? Of area? Of volume?

cubic feet $\qquad$ $cm^2$ $\qquad$ meter

7. (a) Let $f(x) = x^2 - 2x + 1$ and $g(x) = 1 - 3x$. Find both $g(f(x))$ and $f(g(x))$.

(b) Find functions $f$ and $g$ such that $f(g(x)) = \sqrt[3]{x^2 - 1}$.

8. Graph the rational function $g(x) = \dfrac{(x^2 - 1)(x + 2)}{(2x - 1)(x + 3)(x + 2)}$ in the space below.




If any of the following do not exist, so state:

$x$-intercept(s): __________

$y$-intercept(s): __________

Equation(s) of any horizontal asymptote(s): __________

Equation(s) of any vertical asymptote(s): __________

Equation(s) of any slant asymptote(s): __________

Puncture point(s): __________

Fill in the blank: as $x \to \infty$, $y \to$ __________

Fill in the blank: as $x \to -3^+$, $y \to$ __________

9. Write an expression (using the variable $x$) to represent each sequence of operations.

(a) take a number, multiply by $2$, then subtract $3$

(b) take a number, subtract $3$, then multiply by $2$

(c) take a number, multiply it by $2$, cube the result, add $1$, then divide by the original number

Write the sequence of operations that is being described by each expression.

(d) $3x - 1$

(e) $2(x+1)^3 - 5$

(f) $\dfrac{x-3}{7} - 1$

10. Let $f(x) = x^2 - 2x + 1$. Evaluate each of the following expressions.

(a) $f(0)$

(b) $f(1) - 2$

(c) $f(f(-1))$

11. Find the domain of the function $g(x) = \frac{1}{\sqrt{x-3}}$. Report your answer using interval notation.

12. Write the equation of the line, in $y = mx+b$ form, that satisfies the given conditions.

(a) slope $3$, passing through the point $(2, -1)$

(b) the horizontal line that crosses the $y$-axis at $2$

(c) the line that is perpendicular to $x - 3y = 5$ and passes through the point $(0, 3)$

13. (Your calculator is needed for parts of this question.)

(a) What is the domain of the function $f(x) = \dfrac{1-3x}{x-2}$ ?

(b) Use your graphing calculator to graph the function $f$ in the window $-1 < x < 3$ and $-15 < y < 10$.

(c) Find the $x$-intercept of the graph.

(d) Use your calculator to estimate a value for $x$ for which $f(x) = 5$. (Zoom, as necessary, to get f(x) within 0.01 of 5.)

14. Estimate (where necessary) each of the following numbers on your calculator. For full credit, each answer must be correct to five decimal places.

(a) $\dfrac{1+\sqrt{2}}{\sqrt[3]{5} - 7}$

(b) $3x^2 - 5x + 1$, where $x = -1.8$

(c) $|1 - 2x|$, where $x = \sqrt{3}$

(d) $(2.03 \times 10^{-9})(-4.1 \times 10^{7})$

# SOLUTIONS

1. There are many possible correct answers for some of these problems, but these are the most obvious ones:

a) $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b) $23,070,000 = 2.307 \times 10^7$

c) $x^2 - y^2 = (x-y)(x+y)$

d) $\frac{x^4 x^{-1}}{(x^2)^3 x} = \frac{x^3}{x^7} = x^{3-7} = x^{-4}$

e) $300\frac{\text{ft}}{\text{sec}} = 300\frac{\text{ft}}{\text{sec}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \approx 204.5\frac{\text{miles}}{\text{hr}}$

f) $7,036 = 70 \cdot 10^2 + 360 \cdot 10^{-1}$

g) $8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{(8^{1/3})^2} = \frac{1}{2^2} = \frac{1}{4}$

h) Use the technique of completing the square:
$x^2 + 2x + 3 = x^2 + 2x + 1 - 1 + 3 = (x+1)^2 + 2$

i) When $x < -\frac{3}{2}$, $2x + 3 < 0$. Thus, $|2x+3| = -(2x+3) = -2x - 3$.

j) $2\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -6 \\ -2 & 2 \end{bmatrix}$

k) $2 + 3i - i^2 + (1-i)(3+4i) = 2 + 3i - (-1) + 3 + 4i - 3i - 4(-1) = 10 + 4i$

l) Use long division of polynomials to get $\frac{x^2-x-1}{x-3} = x + 2 + \frac{5}{x-3}$. Do a "spot-check": when $x = 0$, we have $\frac{x^2-x-1}{x-3} = \frac{0^2-0-1}{0-3} = \frac{1}{3}$; when $x = 0$, we have $x + 2 + \frac{5}{x-3} = 0 + 2 + \frac{5}{0-3} = \frac{6}{3} - \frac{5}{3} = \frac{1}{3}$. They agree when $x = 0$! (A "spot-check" like this catches lots of mistakes.)

m) Use the change of base formula for logarithms: $\log_b x = \frac{\log_a x}{\log_a b}$
Thus, $\log_7 5 = \frac{\ln 5}{\ln 7}$. Check that $7^{(\log_7 5)} = 5$.

n) $\dfrac{4 \log 10^x}{3} = \dfrac{4}{3} x \log 10 = \dfrac{4}{3} x(1) = \dfrac{4}{3} x$

o) Use properties of logarithms:
$\ln x^4 - \ln x^2 + \ln(x^2+1) = \ln \frac{x^4}{x^2} + \ln(x^2+1) = \ln x^2 + \ln(x^2+1) = \ln x^2(x^2+1)$

p) Use the row of Pascal's triangle beginning with "1 4": Thus, $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$. Since $(x-2y)^4 = (x + (-2y))^4$, we apply this formula with $a = x$ and $b = -2y$ to get:

$$\begin{aligned} (x + (-2y))^4 &= x^4 + 4x(-2y)^3 + 6x^2(-2y)^2 + 4x^3(-2y) + (-2y)^4 \\ &= x^4 - 32xy^3 + 24x^2y^2 - 8x^3y + 16y^4 \end{aligned}$$

q) $(-\infty, -2] \cap (-4, 5] = (-4, -2]$

r) $\{x \mid x \geq -2\} = [-2, \infty)$

2.

$$\text{(a)} \quad 3x(1-5x)(x^2-16)=0$$
$$x=0 \quad \text{or} \quad 1-5x=0 \quad \text{or} \quad x^2-16=0$$
$$x=0 \quad \text{or} \quad x=\frac{1}{5} \quad \text{or} \quad x=\pm 4$$
$$\text{Solution set: } \{0, \frac{1}{5}, 4, -4\}$$

$$\text{(b)} \quad \frac{1}{2}x-7=3x+\frac{x}{5}$$
$$5x-70=30x+2x \quad \text{(clear fractions; multiply by 10)}$$
$$-70=27x$$
$$x=\frac{-70}{27}$$
$$\text{Solution set: } \{-\frac{70}{27}\}$$

$$\text{(c)} \quad |2x-3|>5$$
$$2x-3>5 \quad \text{or} \quad 2x-3<-5$$
$$2x>8 \quad \text{or} \quad 2x<-2$$
$$x>4 \quad \text{or} \quad x<-1$$
$$\text{Solution set: } (-\infty,-1)\cup(4,\infty)$$

$$\text{(d)} \quad 2<|x|<3$$

solve by inspection; want all #s whose distance from 0 is between 2 and 3

$$-3<x<-2 \quad \text{or} \quad 2<x<3$$
$$\text{Solution set: } (-3,-2)\cup(2,3)$$

$$\text{(e)} \quad 1-2x\le 3 \quad \text{or} \quad -3\le x<-2$$
$$x\ge -1 \quad \text{or} \quad -3\le x<-2$$
$$\text{Solution set: } [-3,-2)\cup(-1,\infty)$$

$$\text{(f)} \quad x^2=x+2$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x=2 \quad \text{or} \quad x=-1$$
$$\text{Solution set: } \{-1,2\}$$

(g) $\sqrt{3x^2+5x-3} = x$

square both sides; must check for extraneous solutions at the end

$$3x^2+5x-3 = x^2$$

Solve using the quadratic formula to get:

$$x = \frac{1}{2} \quad \text{or} \quad x = -3$$

Discard $x = -3$; it is an extraneous solution.

Verify that $x = \frac{1}{2}$ is indeed a solution.

Solution set: $\{\frac{1}{2}\}$

(h) $y = x^2 + 1$ and $y = 2x + 4$

A quick sketch verifies that there are two solutions:

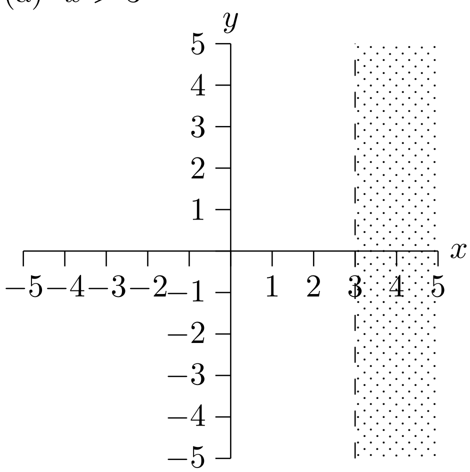
$$x^2 + 1 = 2x + 4$$

$$x = 3 \quad \text{or} \quad x = -1$$

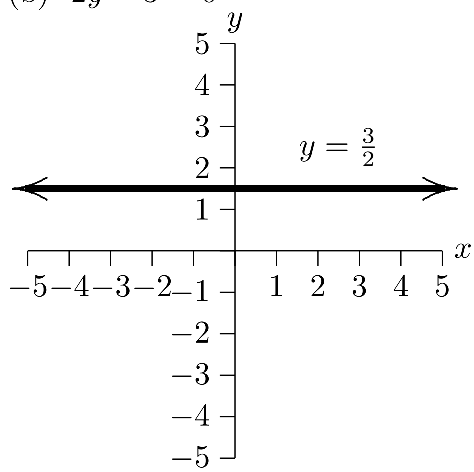
When $x = 3$, $y = 10$; when $x = -1$, $y = 2$.

Solution set: $\{(3, 10), (-1, 2)\}$

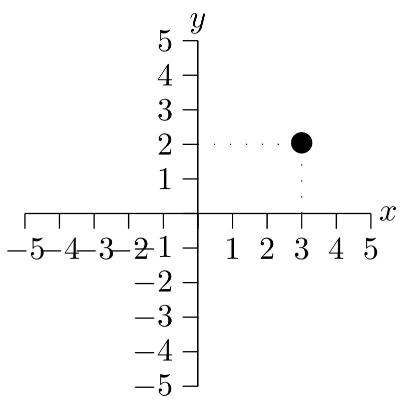
3. (a) $x > 3$



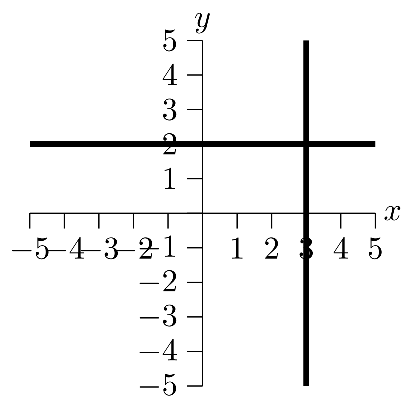
(b) $2y - 3 = 0$



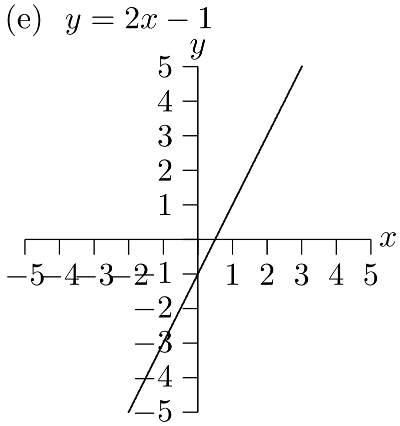

(c) $x = 3$ and $y = 2$



(d) $x = 3$ or $y = 2$

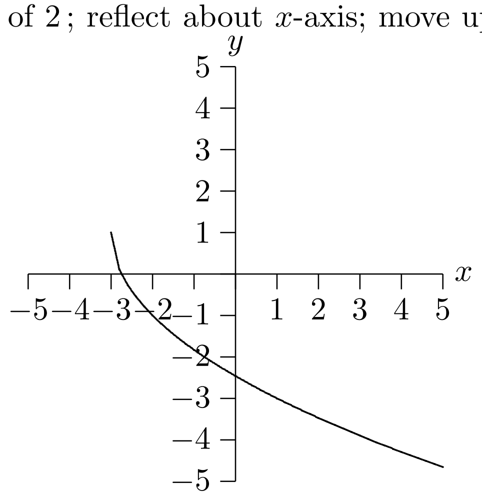


(e) $y = 2x - 1$

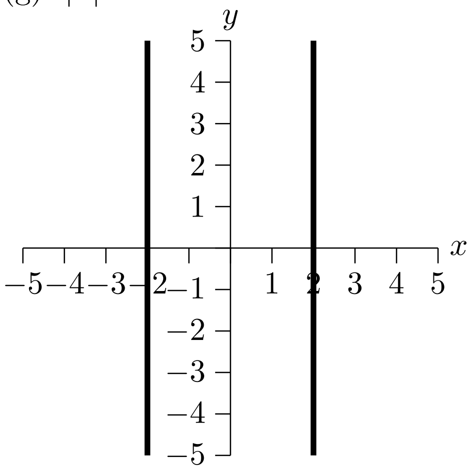


(f) $y = -2\sqrt{x+3} + 1$

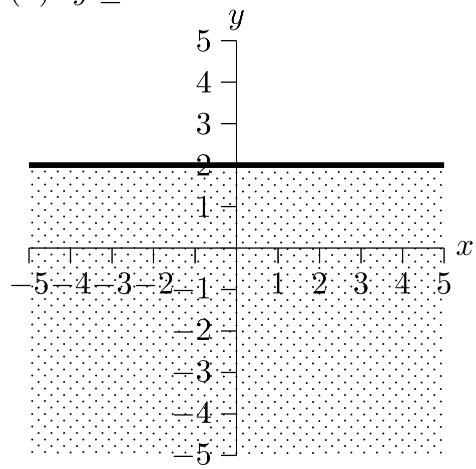
Take $y = \sqrt{x}$, and apply the following transformations: shift left $3$; vertical stretch by a factor of $2$; reflect about $x$-axis; move up 1. This gives:



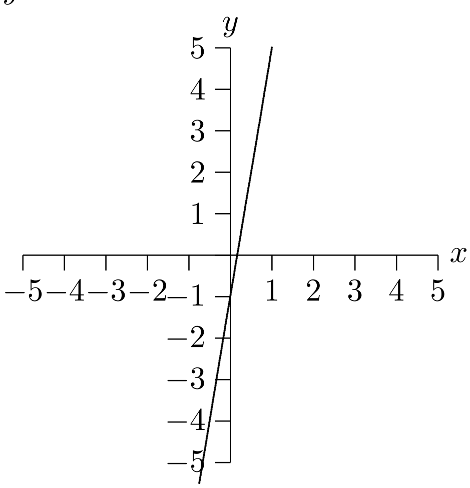
(g) $|x| = 2$



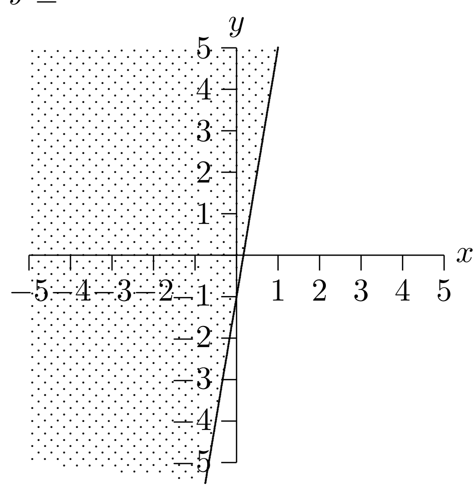

(h) $y \leq 2$



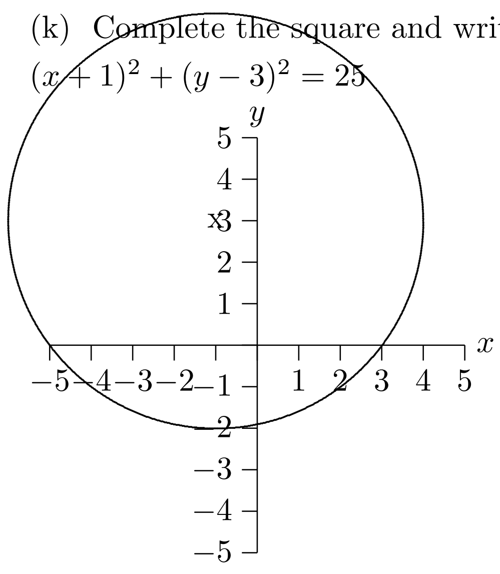

(i) $\frac{y-2}{3} = 2x - 1$ is equivalent to $y = 6x - 1$




(j) $\frac{y-2}{3} \geq 2x - 1$ is equivalent to $y \geq 6x - 1$




(k) Complete the square and write as: $(x + 1)^2 + (y - 3)^2 = 25$



4.

| EQUATION | TRANSFORMATION |
| --- | --- |
| $y = f(x)$ | (starting place) |
| $y = f(x+1)$ | replace $x$ by $x+1$; shift left 1 |
| $y = \lvert f(x+1)\rvert$ | take absolute value of $y$-values; any part below $x$-axis flips up |
| $y = 3\lvert f(x+1)\rvert$ | multiply previous $y$-values by 3; vertical stretch |
| $y = -3\lvert f(x+1)\rvert$ | multiply previous $y$-values by $-1$; reflect about $x$-axis |
| $y = -3\lvert f(x+1)\rvert + 5$ | add 5 to previous $y$-values; move up 5 |

5.

| EQUATION | TRANSFORMATION |
| --- | --- |
| $y = x^2 - 2x + 1$ | (starting place) |
| $y = x^2 - 2x + 2$ | up 1 |
| $y = (x+3)^2 - 2(x+3) + 2$ | left 3 |
| $y = -(x+3)^2 + 2(x+3) - 2$ | reflect about the $x$-axis |
| $y = -2(x+3)^2 + 4(x+3) - 4$ | vertical scale by a factor of 2 |

6. PERIMETER $= 2\ell + 2w$, AREA $= \ell w$

PERIMETER $= 2x + y$, AREA $= \frac{1}{2}(y)(\frac{x}{2}) = \frac{1}{4}xy$

CIRCUMFERENCE $= 2\pi r$, AREA $= \pi r^2$

VOLUME = (area of base)(height) $= \pi R^2 h$

Meter is a unit of length; $\text{cm}^2$ is a unit of area; cubic feet is a unit of volume.

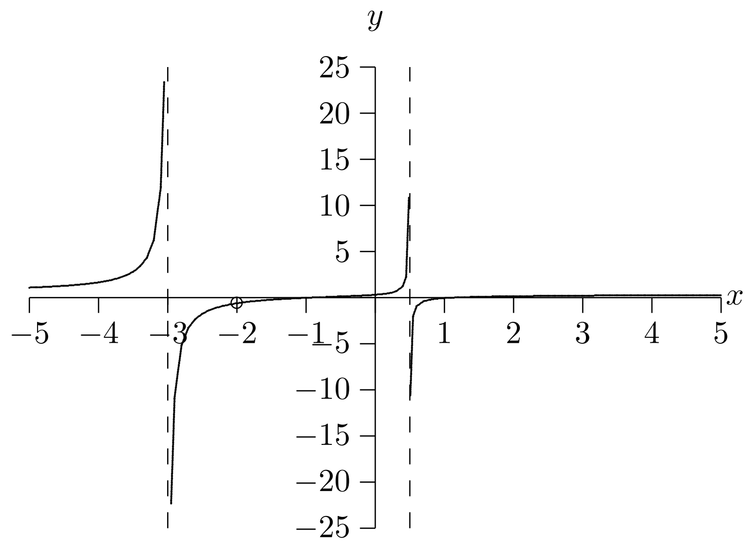
7.

$$
\begin{aligned}
\text{(a)}\quad g(f(x)) &= g(x^2 - 2x + 1) \\
&= 1 - 3(x^2 - 2x + 1) \\
&= 1 - 3x^2 + 6x - 3 \\
&= -3x^2 + 6x - 2
\end{aligned}
$$

$$
\begin{aligned}
f(g(x)) &= f(1 - 3x) \\
&= (1-3x)^2 - 2(1-3x) + 1 \\
&= 1 - 6x + 9x^2 - 2 + 6x + 1 \\
&= 9x^2
\end{aligned}
$$

(b) (There are other possible correct answers.) Let $g(x) = x^2 - 1$ and $f(x) = \sqrt[3]{x}$.

8.




For $x \neq -2$, $g(x) = \dfrac{x^2 - 1}{(2x-1)(x+3)}$.

Note that the point $(-2, -\frac{3}{5})$ is a puncture point.

$x$-intercepts occur when $x = \pm 1$.

$y$-intercept: $(0, \frac{1}{3})$

horizontal asymptote: $y = \frac{1}{2}$

vertical asymptotes: $x = \frac{1}{2}$ and $x = -3$

no slant asymptote

As $x \to \infty$, $y \to \frac{1}{2}$.

As $x \to -3^+$, $y \to -\infty$.

9. (a) $2x - 3$

(b) $2(x-3)$

(c) $\frac{(2x)^3+1}{x}$

(d) take a number, multiply by 3, then subtract 1

(e) take a number, add 1, cube the result, multiply by 2, then subtract 5

(f) take a number, subtract 3, divide by 7, then subtract 1

10. (a) $f(0) = 0^2 - 2(0) + 1 = 1$

(b) $f(1) - 2 = \big(1^2 - 2 \cdot 1 + 1\big) - 2 = 0 - 2 = -2$

(c) $f(f(-1)) = f\big((-1)^2 - 2(-1) + 1\big) = f(4) = 4^2 - 2(4) + 1 = 9$

11. The function $g$ is defined whenever $x - 3 > 0$, that is, whenever $x > 3$.

The domain of $g$ is the interval $(3, \infty)$.

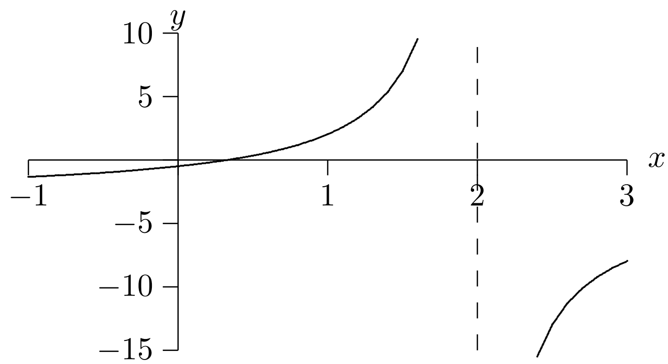
12. (a) $y = 3x - 7$

(b) $y = 2$

(c) The line $x - 3y = 5$ has slope $\frac{1}{3}$; a perpendicular line will have slope $-3$.
The line with slope $-3$ passing through $(0, 3)$ has equation $y = -3x + 3$.

13. (a) The domain of $f$ is the set of all real numbers except $2$.

(b)




(c) The graph crosses the $x$-axis at $\frac{1}{3}$. (Set $1 - 3x = 0$. Be sure you can get this *exact* answer, not just $x \approx 0.333333$.)

(d) When $x = 1.375$ (exactly), then $f(x) = 5$. (You could check this, if desired, by solving the equation $5 = \frac{1-3x}{x-2}$.)

14. (a) $\frac{1+\sqrt{2}}{\sqrt[3]{5}-7} \approx -0.45637$

(b) 19.72 (this is exact)

(c) $|1 - 2\sqrt{3}| \approx 2.46410$

(d) $(2.03 \times 10^{-9})(-4.1 \times 10^{7}) = -0.08323$ (this is exact)