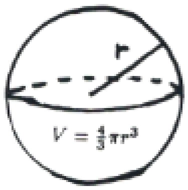
2.1 Functions and Function Notation

Introduction

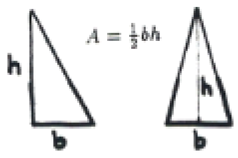
inputs and outputs

In many common relationships between variables, there are natural input/output roles assumed by the variables.

For example, in the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere, one naturally thinks of 'inputting' a radius r into the formula, and 'outputting' the volume of the sphere with that radius.



Although the radius is 'naturally' viewed as the input when the formula is written in the form $V = \frac{4}{3}\pi r^3$, this need not always be the case. Suppose, for example, that spherical containers are being designed to hold various amounts of liquid. Given a desired volume, it is necessary to determine the radius of the sphere that will yield that volume. In this case, one can solve for r, yielding the equivalent equation $r = \sqrt[3]{\frac{3V}{4\pi}}$. In this formulation of the equation, the volume V is now viewed as the input, and the radius r as the output.



As a second example, in the formula $A = \frac{1}{2}bh$ for the area of a triangle, one naturally thinks of 'inputting' both the base b and height h of the triangle, and 'outputting' the area of a triangle with that base and height. However, the equivalent equation $h = \frac{2A}{b}$ views the area and base as inputs, and the height as an output.

EXERCISE 1

♣ 1. Consider the formula $A = \pi r^2$ for the area of a circle. What is naturally viewed as the input? Output? Rewrite the equation so that the radius is the 'natural' output.

♣ 2. Come up with a common relationship between variables (different from the example above) that has two inputs.

♣ 3. Come up with a common relationship between variables that has three inputs.

functions

The language of mathematics provides a very precise tool for discussing this kind of input-output relationship between variables: functions. Functions, and the notation used in connection with functions, is the topic of this section.

function, informal definition

A *function* is a special relationship between variables, where to each choice of input there corresponds *a unique* output. In this case, we say that 'the output *is a function of* the input'.

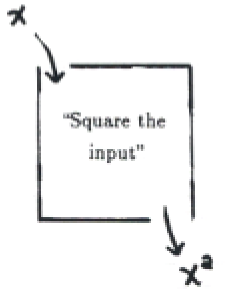
The most important word in this informal definition of function is the word 'unique'. The examples in this section should clarify the importance of the uniqueness of the output.

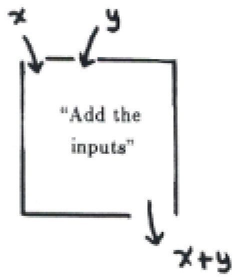
For example, in the formula $V = \frac{4}{3}\pi r^3$, to each value of input r there corresponds a unique volume V. We say that V *is a function of* r.

In the formula $A = \frac{1}{2}bh$, to each choice of base b and height h there corresponds a unique area A. We say that A *is a function of* b *and* h. (Here, we can view the input as an ordered pair of numbers, (b, h).)

functions as ‘black boxes’

It is often helpful to think of a function in terms of a ‘black box’. The input goes in the top of the box. The box itself is the ‘rule’ that does something to the input. The output drops out the bottom of the box.





EXERCISE 2

♣ 1. In the first box pictured above, what will the output be if the input is 5? -5? x? x^2? $x+h$?

♣ 2. In the second box pictured above, what will the output be if the inputs are 3 and 4? What if the inputs are x^2 and y^2? What if the inputs are t and t?

What should you think when you hear the phrase: ‘y is a function of x’?

When you hear the phrase *y is a function of x*, you can roughly think: *y depends on x*. More precisely, think: *y is an output that is uniquely determined by the input x*.

functions often arise naturally from equations

In the previous chapter, we studied equations. Many (but not all) equations describe a function relationship between their variables. The form in which the equation is written often leads to a choice of input/output roles for the variables.

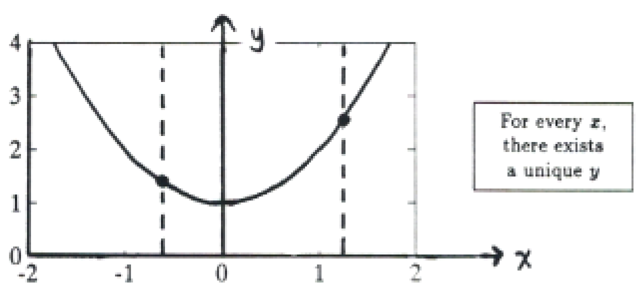
obtaining a function by solving for y in terms of x

For example, suppose that an equation in the variables x and y is such that we are able to solve the equation for y in terms of x, thus obtaining an (equivalent) equation:

$$y = <\textit{ some formula involving } x >$$

In this form, y is naturally viewed as an output that depends on the input x. That is, once a value is chosen for x, we can plug it into the formula, and obtain the unique corresponding value of y. So, y is a function of x.

For example, consider the equation $y - x^2 = 1$. This is equivalent to $y = x^2 + 1$; once an input x is chosen, the unique output y is found by squaring x, then adding 1. So, y is a function of x. The graph of $y = x^2 + 1$ is shown below.

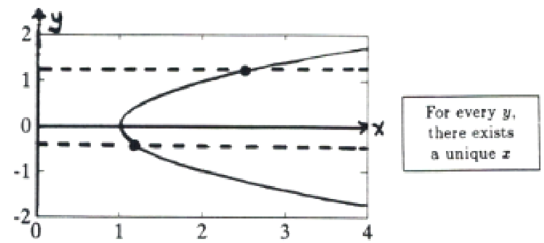


vertical line test;
y is a function of x

There is a nice graphical way to see that every input x corresponds to a unique output y. Imagine a vertical line sweeping through all the x-values. No matter what x-value we 'stop' this vertical line at, it hits exactly one point on the graph—the unique y-value associated with that x-value.

horizontal line test;
x is a function of y

Now reverse the roles of x and y in the previous example. That is, consider the equation $x - y^2 = 1$, which is equivalent to $x = y^2 + 1$, and has the graph shown below. Associated to each 'input' y there is a unique 'output' x, so in this case, x is a function of y.



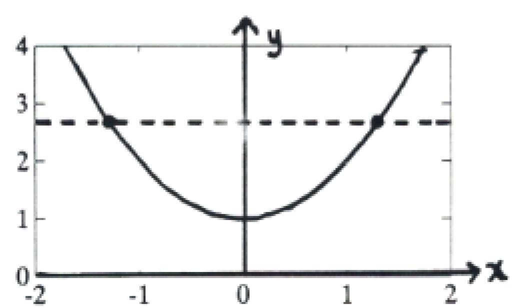
How can we graphically check that x is a function of y? We must check that each allowable y-value is associated with a *unique* x-value. To do this, imagine a horizontal line sweeping through the graph, checking each y-value. If this horizontal line never hits the graph at more than one point, then x is a function of y.

EXAMPLE
y is a function of x
but
x is not a function of y

Once more consider the equation $y - x^2 = 1$. Suppose it is desired to view x as the 'output'; in this case, we are motivated to solve for x (see the Algebra Review, this section) giving:

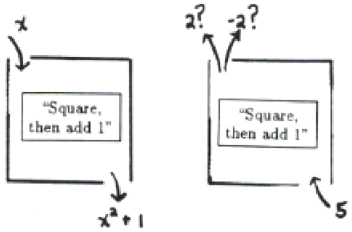
$$\begin{aligned} y - x^2 = 1 \quad &\iff \quad x^2 = y - 1 && \text{(add } x^2\text{, subtract 1, rearrange)} \\ &\iff \quad x = \pm\sqrt{y-1} && \text{(take square roots, correctly!)} \end{aligned}$$

Now, corresponding to an allowable 'input' y, we obtain *two* 'outputs': $x = +\sqrt{y-1}$ and $x = -\sqrt{y-1}$. In particular, we do not obtain a *unique* output value. Thus, although y is a function of x in this equation; x is not a function of y. Note that the graph of $y - x^2 = 1$ does not pass a horizontal line test.

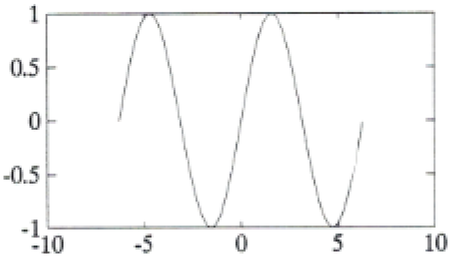


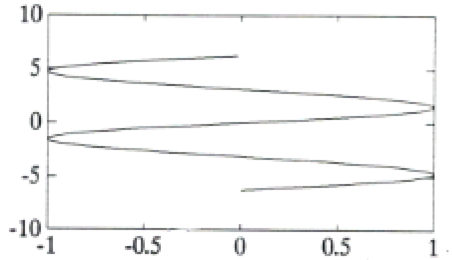
'black box' interpretation of the previous example

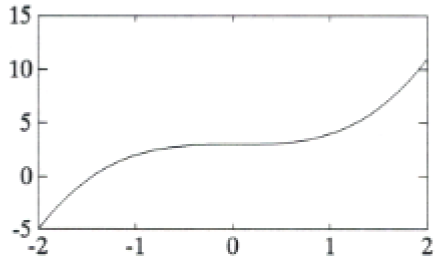
It may be helpful to interpret this example in terms of a 'black box'. Since y is a function of x, we can drop any input into the 'square, then add 1' box, and a unique output will drop out the bottom. However, since x is not a function of y, we can *not* necessarily reverse this process. That is, suppose we pick up the number 5 from the output pile. Can we put it in the box (backwards) to determine where it came from? The answer is no: both 2 and -2 gave rise to the output 5.



EXERCISE 3 ♣ Consider the graphs shown below. Which ones describe y as a function of x? Which ones describe x as a function of y?







EXAMPLE

an equation in x and y with no function relationships between the variables

Consider the equation $x^2 + y^2 = 9$. Solving for y yields:

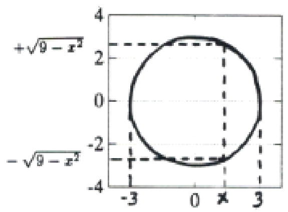
$$\begin{aligned} x^2 + y^2 = 9 &\iff y^2 = 9 - x^2 && \text{(subtract } x^2\text{)} \\ &\iff y = \pm\sqrt{9 - x^2} && \text{(take square roots, correctly!)} \end{aligned}$$

What are the allowable values to choose for x? The expression under the square root must be nonnegative:

$$\begin{aligned} 9 - x^2 \ge 0 &\iff 9 \ge x^2 && \text{(add } x^2 \text{ to both sides)} \\ &\iff x^2 \le 9 && \text{(rearrange)} \\ &\iff |x| \le 3 && \text{(take square roots—correctly!)} \\ &\iff -3 \le x \le 3 && \text{(solve the abs. value inequality)} \end{aligned}$$

Given an appropriate value of x ($x \in [-3, 3]$), we get *two* associated values of y: $+\sqrt{9 - x^2}$ and $-\sqrt{9 - x^2}$. So in this case, y is not a function of x. The graph of $x^2 + y^2 = 9$ is shown below. Observe that it fails the vertical line test.

The exercise below completes this example.



EXERCISE 4

♣ 1. Solve the equation $x^2 + y^2 = 9$ for x. Be sure to write a complete mathematical paragraph.

♣ 2. What are the allowable values for y? Be sure to write a complete mathematical paragraph when finding them.

♣ 3. Is x a function of y?

★

global vs. local

★ At most points on the graph of $x^2 + y^2 = 9$, y is *locally* a function of x, in the following sense. Let (a, b) be a point on the graph with $a \ne 3$ and $a \ne -3$. Then, there exists an interval I containing a such when the graph is restricted to x-values in this interval, y is a function of x. This observation becomes important in the section on *implicit differentiation*.

EXAMPLE

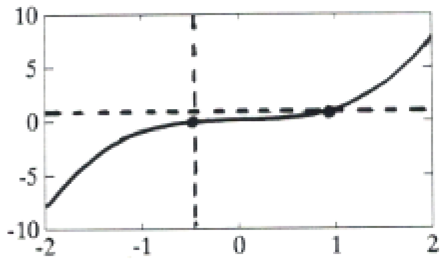
y is a function of x and x is a function of y

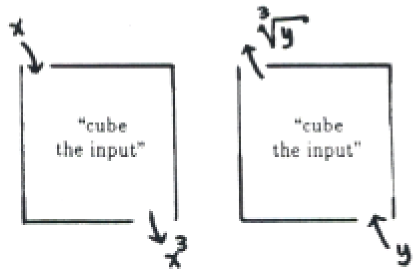
Consider the equation $y = x^3$. Given an input x, there is a unique corresponding output y, obtained by cubing x. So, y is a function of x.

If we choose to view y as the input, we can rewrite the equation in the equivalent form $x = \sqrt[3]{y}$. Now, given an 'input' y, there is a unique corresponding 'output' x, obtained by taking the cube root of y. So, x is also a function of y.

The graph of $y = x^3 \iff x = \sqrt[3]{y}$ is shown below. Observe that it passes both a vertical line test *and* a horizontal line test.

Here's the 'black box' interpretation of this example. When an input is dropped in the top of the 'cube' box, a unique output drops out the bottom. If we pick up a number from the output pile, we can use the box 'backwards' to obtain the unique input from which it came. That is, associated to every input is a unique output; and associated to every output is a unique input. This type of relationship between x and y is particularly nice.





EXERCISE 5

Consider the equation $x = |y|$.

♣ 1. Graph this equation.

♣ 2. Is y a function of x? Why or why not?

♣ 3. Is x a function of y? Why or why not?

EXERCISE 6

Consider the equation $y = 3$, viewed as an equation in two variables, x and y.

♣ 1. Graph this equation.

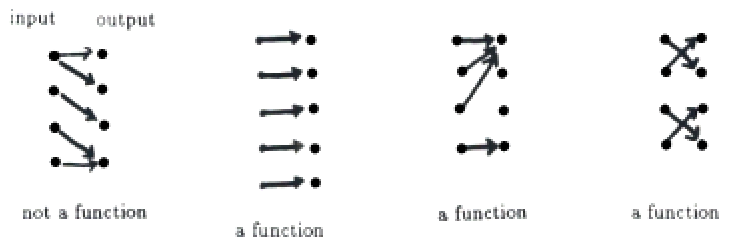
♣ 2. Is y a function of x? Why or why not?

♣ 3. Is x a function of y? Why or why not?

♣ 4. What 'black box' would you associate with the equation $y = 3$? In particular, what is the 'rule' that the black box performs in this case?

mapping diagrams

It is sometimes helpful to view functions/non-functions in terms of mapping diagrams, as illustrated below.



notation for functions

Next, we introduce an *extremely important* notation used in connection with functions.

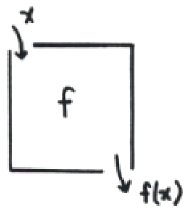
FUNCTION NOTATION

x, the input
f, the rule
$f(x)$, the output corresponding to the input x

In a function relationship between variables, once an input is chosen, there is a unique corresponding output. It is convenient to give a name to the output that illustrates its relationship to the input.

Here's how it's done: if the input is x, the output is called $f(x)$ (read as 'f *of* x'). Letters other than f are possible. The letter f is merely common because it is the first letter in the word 'function'.

The sketch below illustrates the relationship between the function f, the input x, and the output $f(x)$.



Until you become an expert on functions, it is important that you distinguish between the *function* f (the 'rule'), and the output $f(x)$ that comes from the input x. Unfortunately, f and $f(x)$ are often used synonymously, leading to confusion.

Correct: *The function f is the squaring function.*
Incorrect: *The function $f(x)$ is the squaring function.*

naming functions

The function that takes an input and squares it can be described in function notation in any of the following ways:

$$f(x) = x^2 \quad \text{or} \quad g(x) = x^2 \quad \text{or} \quad h(x) = x^2 \quad \text{or} \quad S(x) = x^2$$

It is often good to choose a letter name for the function that helps to describe the function; in this sense, perhaps '$S(x) = x^2$' is good, because 'S' is the first letter in the word 'square'.

If $S(x) = x^2$, then what is $S(3)$? Answer: $S(3) = 3^2 = 9$. '$S(3)$' is the name given to the unique output when 3 is the input.

What is $S(x + y)$? Answer: $S(x + y) = (x + y)^2$. Be sure to write a complete sentence for the answer: don't just say '$(x + y)^2$'.

What is $S(x^2)$? Answer: $S(x^2) = (x^2)^2 = x^4$.

What is $S(\square)$? Answer: $S(\square) = (\square)^2$. (Fill in the box with any input you want!)

dummy variables

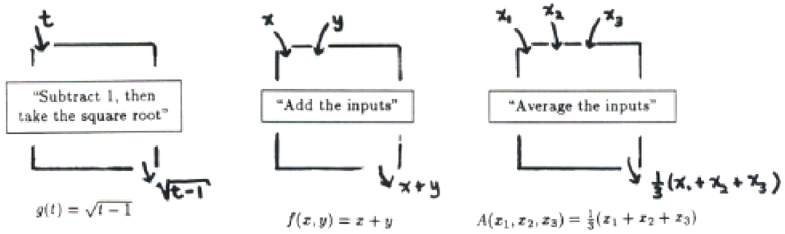
Here are some more ways that the squaring function can be described:

$$S(y) = y^2 \quad \text{or} \quad S(t) = t^2 \quad \text{or} \quad S(\alpha) = \alpha^2 \quad \text{or} \quad S(\omega) = \omega^2$$

The variable in parentheses after the function name represents a typical input; you may give any name you want to this input. Then, the formula on the right-hand side tells you what the function (the 'rule') does to this input. Since lots of different names can be used to express the same information, this input variable is called a *dummy variable*. Try to choose an appropriate name for the dummy variable. One common name is 'x'. However, if the inputs represent time values, 't' is probably more appropriate.

EXAMPLE

The 'black boxes' below illustrate the use of function notation.

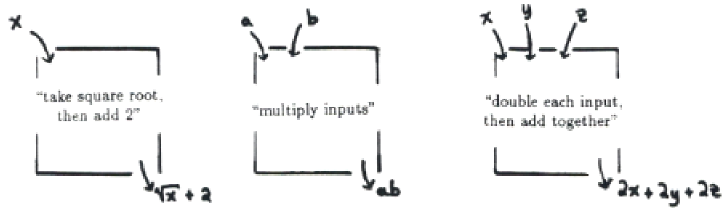


EXERCISE 7

♣ 1. Describe, in function notation, the rule: 'take a number, double it, then add 3.' Express this same rule in three different ways. What is the output if $x + y$ is the input?

♣ 2. Describe, in function notation, the rule: 'take 2 numbers and average them'. Do this in three different ways. What is the output if x and $5x$ are the inputs?

EXERCISE 8 ♣ Write function notation that corresponds to the 'black boxes' shown below.



function of 1 *variable*
function of 2 *variables*

A function like $f(x) = x^2 - 1$ that takes one input is called a *function of one variable.*

A function like $f(x, y) = 2x - 3y$ that takes two inputs is called a *function of two variables.*

♣ What do you suspect a function like $f(x, y, z) = x + y + z$, that takes three inputs, is called?

practice with function notation

Here's some practice with function notation:

Consider the function g given by $g(x) = x^2 + 3$. Then:

$$\begin{aligned}
g(0) &= 0^2 + 3 = 3 \\
g(3) &= 3^2 + 3 = 12 \\
g(-3) &= (-3)^2 + 3 = 12 \\
g(x + h) &= (x + h)^2 + 3 = x^2 + 2xh + h^2 + 3 \\
g(x^2) &= (x^2)^2 + 3 = x^4 + 3 \\
g(g(t)) &= g(t^2 + 3) = (t^2 + 3)^2 + 3 \quad \text{(inside out)} \\
g(g(t)) &= (g(t))^2 + 3 = (t^2 + 3)^2 + 3 \ \text{(outside in)}
\end{aligned}$$

The last two lines illustrate two different correct paths leading to the same result.

Now consider the function g given by $g(x, y) = x^3 + y$. Then:

$$\begin{aligned}
g(0, 0) &= 0^3 + 0 = 0 \\
g(-1, 2) &= (-1)^3 + 2 = 1 \\
g(2, -1) &= 2^3 + (-1) = 7 \\
g(a, b) &= a^3 + b \\
g(y, x) &= y^3 + x \\
g(f(x), x) &= (f(x))^3 + x \\
g\big(g(a, b), g(c, d)\big) &= (g(a, b))^3 + g(c, d) = (a^3 + b)^3 + (c^3 + d)
\end{aligned}$$

EXERCISE 9

Consider the function d given by $d(x) = \frac{f(x+h)-f(x)}{h}$. Here, f is a function of one variable, and h is a constant.

Find the following. Write complete mathematical sentences.

♣ 1. $d(0)$

♣ 2. $d(y)$

♣ 3. $d(x+h)$

♣ 4. $d(x^2)$

Now, define D by $D(x,h) = \frac{f(x+h)-f(x)}{h}$. Find the following:

♣ 5. $D(0,1)$

♣ 6. $D(y,k)$

♣ 7. $D(x+\epsilon,k)$

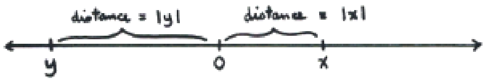
♣ 8. $D(x+h,2h)$

ALGEBRA REVIEW

absolute value, geometric definition, set union $\cup$

absolute value

Let x denote any real number. The *absolute value of* x, denoted $|x|$, is its distance from 0 on the number line.



EXAMPLE

solving a simple absolute value equation

Problem: Solve the absolute value equation $|x| = 3$.

Solution: Think of this as follows: We seek all numbers x whose distance from 0 on the number line is equal to 3. There are two such numbers, 3 and -3. Thus, the solution set of the equation $|x| = 3$ is $\{3, -3\}$. One commonly writes:

$$|x| = 3 \iff x = \pm 3$$

where '$x = \pm 3$' is read as 'x *equals plus or minus* 3', and means '$x = 3$ or $x = -3$'.

EXERCISE 10

Consider the sentence:

$$x = 3 \text{ or } x = -3 \qquad (*)$$

♣ 1. Let x be 3. For this choice of x, is (*) true or false? (If necessary, review the mathematical meaning of the word 'or'.)

♣ 2. What is the solution set of (*)?

♣ 3. Let x be 4. For this choice of x, is (*) true or false?

EXAMPLE

solving a simple absolute value inequality

Solve the absolute value inequality $|y| > 2$. Here, we seek all real numbers y whose distance from 0 is greater than 2. Since we can walk both *to the right* and *to the left* on the number line, the solution set has two pieces, and can be expressed in several different ways:

solution set of $|y| > 2 =$ 

$= (2, \infty) \cup (-\infty, -2)$ (the symbol '$\cup$' is discussed below)

$= \{y \mid y > 2 \text{ or } y < -2\}$

$= \{y \mid y > 2\} \cup \{y \mid y < -2\}$

$\cup$, set union

Here, the symbol $\cup$ has been used to denote the operation of *set union*. For sets A and B, a new set $A \cup B$ (read as 'A union B') is formed by 'throwing together' all the elements of both sets. Precisely:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

For example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ then $A \cup B = \{1, 2, 3, 4, 5\}$.

EXERCISE 11

Write the following sets in as simple a way as possible:

♣ 1. $(-1, 0) \cup [0, 2)$

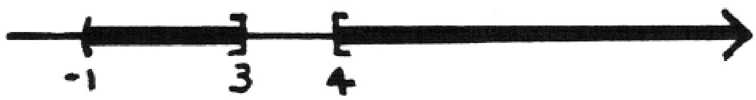
♣ 2. $\{x \mid x \text{ is rational}\} \cup \{x \mid x \text{ is irrational}\}$

♣ 3. $\{-1, 2, 100\} \cup \mathbb{Z}$

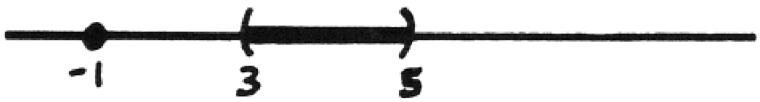
♣ 4. $\{t \mid t \geq 0\} \cup \{x \mid x < 0\}$

Write the following sets, using correct set notation and the $\cup$ symbol (if appropriate).

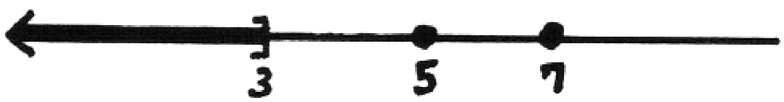
♣ 5.



♣ 6.



♣ 7.



EXERCISE 12

Solve the following absolute value equations/inequalities. Be sure to write complete and correct mathematical sentences. Show the solution sets on a number line.

♣ 1. $|x| < 2$ (Hint: What numbers have a distance from 0 that is less than 2?)

♣ 2. $|t| \ge 3$

♣ 3. $5 - |x| = 1$

♣ 4. $2 - |t| < -3$

Write an absolute value equation or inequality whose solution set is the set of numbers shown:

♣ 5. 

♣ 6. 

♣ 7. 

$|x| = \sqrt{x^2}$

Here is a characterization of the absolute value that is particularly useful in many situations:

$$\text{For all real numbers } x,\ \ |x| = \sqrt{x^2}.$$

For example, $\sqrt{(-5)^2} = |-5| = 5$, *not* -5! (Remember what $\sqrt{x^2}$ represents; the *nonnegative* number which, when squared, yields x^2.)

taking the square root of both sides of an equation

We have learned that adding the same number to both sides of an equation does not change its solution set; and multiplying both sides by any nonzero number doesn't change its solution set. How about taking the square root of both sides? Answer: *Providing you take the square root correctly, you WILL get an equivalent equation.*

take the square root incorrectly; lose a solution

Unfortunately, many students *don't* take the square root correctly, and write down things like this:

$$x^2 = 9$$
$$\sqrt{x^2} = \sqrt{9}$$
$$x = 3$$

This student has *lost a solution.* The only time that $\sqrt{x^2}$ equals x, is if x happens to be nonnegative. The original equation $x^2 = 9$ has solution set $\{3, -3\}$; the final equation $x = 3$ only has solution 3.

taking the square root correctly

If the student had instead used the fact that is correct for *all* real numbers, $\sqrt{x^2} = |x|$, the solution would not be lost. Writing a complete mathematical sentence this time, we get:

$$\begin{aligned} x^2 = 9 \quad &\iff \quad \sqrt{x^2} = \sqrt{9} \\ &\iff \quad |x| = 3 \\ &\iff \quad x = \pm 3 \end{aligned}$$

It is conventional to leave out the step '$\sqrt{x^2} = \sqrt{9}$', and go directly to '$|x| = 3$'.

The next theorem makes this idea precise. As you read it, be thinking: what do these facts tell me that I can DO?

THEOREM

taking square roots of equations and inequalities

For all real numbers z, and for $a \ge 0$:

$$\begin{aligned} z^2 = a &\iff |z| = \sqrt{a} \iff z = \pm\sqrt{a} \\ z^2 > a &\iff |z| > \sqrt{a} \iff z > \sqrt{a} \text{ or } z < -\sqrt{a} \\ z^2 < a &\iff |z| < \sqrt{a} \iff -\sqrt{a} < z < \sqrt{a} \end{aligned}$$

here's how an instructor might translate this theorem

This is the way an instructor might 'translate' (part of) this theorem for students: 'You can take the square root of both sides of an equation $z^2 = a$ providing that you use the correct formula for $\sqrt{z^2}$; that is, $\sqrt{z^2} = |z|$.'

Here's a typical use of this theorem, where 'z' is '$x - 1$':

Problem: Solve $(x-1)^2 = 5$.

Solution:

$$\begin{aligned} (x-1)^2 = 5 &\iff |x-1| = \sqrt{5} \\ &\iff x - 1 = \pm\sqrt{5} \\ &\iff x = 1 \pm \sqrt{5} \end{aligned}$$

The solutions of '$(x-1)^2 = 5$' are $1 \pm \sqrt{5}$. ♣ Check!

EXERCISE 13

Use the previous theorem to solve the following equations/inequalities. Be sure to write down complete mathematical sentences.

♣ 1. $t^2 = 7$

♣ 2. $(2t-5)^2 = 3$

♣ 3. $x^2 < 4$

♣ 4. $x^2 + 6x + 9 > 4$ (Hint: Factor the left-hand side.)

♣ 5. $(|t| - 2)^2 < 1$

EXERCISE 14

♣ 1. Write a theorem, the way a mathematician would, that says how to go about solving an inequality of the form $z^2 \ge a$ (for $a \ge 0$). Then, use your theorem to solve $(2x-1)^2 \ge 3$. Show your solution set on a number line.

♣ 2. Write a theorem, the way a mathematician would, that says how to go about solving an inequality of the form $z^2 \le a$ (for $a \ge 0$). Then, use your theorem to solve $(2x-1)^2 \le 3$. Show the solution set on a number line. Compare your answer with the previous problem—do you believe your result?

★

An astute student may have noticed that the previous theorem does not apply to a sentence like $y^2 = 9 - x^2$, since the right-hand side is not nonegative for all values of x. Indeed, the sentences '$y^2 = 9 - x^2$' and '$y = \pm\sqrt{9 - x^2}$' have different implied domains.

However, if x and y are any real numbers that make $y^2 = 9 - x^2$ TRUE, then $9 - x^2$ must be nonnegative (since it equals y^2, which is nonnegative). Then, it must also be true that $y = \pm\sqrt{9 - x^2}$.

And, if x and y are any real numbers that make $y = \pm\sqrt{9 - x^2}$ TRUE, then, the sentence $y^2 = 9 - x^2$ must also be true.

Thus, the sentences $y^2 = 9 - x^2$ and $y = \pm\sqrt{9 - x^2}$ do indeed have identical graphs.

EXERCISE 15

♣ 1. Solve the equation $y^2 - x^2 = 1$ for y in terms of x. Be sure to write a complete and correct mathematical sentence.

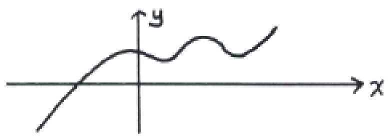
♣ 2. Solve the equation $y^2 - x^2 = 1$ for x in terms of y. Be sure to write a complete and correct mathematical sentence.

♣ 3. If (x, y) is a pair of real numbers that makes the sentence '$y^2 - x^2 = 1$' true, what (if anything) can be said about y? (Hint: Look at your solution to (2).)

QUICK QUIZ

sample questions

1. In the graph shown, is y a function of x? Is x a function of y? Justify your answers.



2. Solve the equation $x^2 - y + 1 = 0$ for y. Be sure to write a complete mathematical sentence. Is y a function of x?
3. Solve the equation $x^2 - y + 1 = 0$ for x. Be sure to write a complete mathematical sentence. Is x a function of y?
4. Describe, in function notation, the rule: 'take a number, halve it, subtract 3, then square the result'.
5. Let $g(x) = 2x^2 - 1$. Find $g(-1)$; find $g(x^2)$.

KEYWORDS *for this section*	*Inputs, outputs, function, 'black box' view of functions, vertical line test, horizontal line test, absolute value of x, solving absolute value equations, solving absolute value inequalities, solving equations of the form $z^2 = a$, solving inequalities of the form $z^2 < a$, $z^2 > a$, $z^2 \le a$, $z^2 \ge a$, set union $(A \cup B)$, $\|x\| = \sqrt{x^2}$, mapping diagrams, function notation, f versus $f(x)$, dummy variable, function of n variables.*

END-OF-SECTION EXERCISES

more practice with function notation

Find the indicated function values.

1. $f(x) = x^3 - 1:\quad f(0),\quad f(1),\quad f(-1),\quad f(t),\quad f(f(2))$
2. $g(x) = -x^4 + x:\quad g(x+h),\quad g(-x),\quad g(-1)$
3. $f(x) = |x|:\quad f(-2),\quad f(t),\quad f(-t),\quad f(x^2)$
4. $g(x) = |x-2|:\quad g(-x),\quad g(|t|),\quad g(\sqrt{2}),\quad g(x+2)$
5. $h(x) = \frac{1}{x}:\quad h(-x),\quad h(h(x)),\quad h(\frac{1}{x}),\quad h(x+\Delta x),\quad h(|x|)$
6. $h(x) = \sqrt{x^2-1}:\quad h(t),\quad h(x+\Delta x),\quad h(-x),\quad h(1)$
7. $h(x,y) = x^2 + y^2 - 1:\quad h(1,1),\quad h(x,x),\quad h(y,x),\quad h(x+\Delta x, y+\Delta y)$
8. $h(x,y) = \frac{1}{x(y-1)}:\quad h(0,0),\quad h(y,x),\quad h(x^2,y),\quad h(x,y^2)$