SOLUTIONS TO EXERCISES:
THESE SENTENCES CERTAINLY LOOK DIFFERENT

IN-SECTION EXERCISES:

1. Webster's New Collegiate Dictionary gives this definition for 'equal': *(1) of the same measure, quantity, amount, or number as another (2) identical in mathematical value or logical denotation: EQUIVALENT*

2. In the Webster's New Collegiate Dictionary definition, the word 'equal' appears four times in the definition of 'equivalent'.

3. In English, the distinction between 'equal' and 'equivalent' is certainly somewhat blurred.

4. In Mathematics, 'equal' and 'equivalent' have very different meanings. They are definitely *not* synonyms!

5. It is being used as a connective for sentences.

The subsentence 'it can be used as a connective for sentences' is true.

The subsentence 'it can be used as a connective for nouns' is true.

Consequently, the compound sentence 'it can be used as a connective for sentences, **and** it can (also) be used as a connective for nouns' is true.

6. (a) TRUE; line 1

(b) FALSE; line 3

(c) TRUE; line 4

(d) FALSE; line 2

7. Either both subsentences are TRUE (line 1) or both subsentences are FALSE (line 4).

8. The subsentences have different truth values: one is true, the other is false (lines 2 or 3 of the truth table).

9. For all real numbers t, $t-4=0$ is equivalent to $t=4$.

For all real numbers t, $t-4=0$ if and only if $t=4$.

For all real numbers t, $t-4=0$ iff $t=4$.

For all real numbers t, $t-4=0 \iff t=4$.

10. Each sentence from (9) gives the same information: no matter what real number is chosen for t, the sentences '$t-4=0$' and '$t=4$' have the same truth values. They are true at the same time, and false at the same time. Indeed, each sentence is true when t is 4, and false otherwise.

11. (a) '$5-t=0$': What number, when subtracted from 5, gives 0? Answer: 5

Thus, the first sentence is true when t is 5, and false otherwise.

'$t=5$': What number is equal to 5? Answer: 5

Thus, the second sentence is true when t is 5, and false otherwise.

These sentences always have the same truth values. The second one is easier to work with.

A mathematician might summarize these observations by saying:

For all real numbers t, $5-t=0$ is equivalent to $t=5$.

For all real numbers t, $5-t=0$ if and only if $t=5$.

For all real numbers t, $5-t=0$ iff $t=5$.

For all real numbers t, $5-t=0 \iff t=5$.

(b) '$3x=12$': What number, when multiplied by 3, gives 12? Answer: 4

Thus, the first sentence is true when x is 4, and false otherwise.

'$x=4$': What number is equal to 4? Answer: 4

Thus, the second sentence is true when x is 4, and false otherwise.

These sentences always have the same truth values. The second one is easier to work with.

For all real numbers x, $3x = 12$ is equivalent to $x = 4$.

For all real numbers x, $3x = 12$ if and only if $x = 4$.

For all real numbers x, $3x = 12$ iff $x = 4$.

For all real numbers x, $3x = 12 \iff x = 4$.

(c) '$x = 14$': What number equals 14? Answer: 14

Thus, the first sentence is true when x is 14, and false otherwise.

'$\frac{x}{2} = 7$': What number, when divided by 2, yields 7? Answer: 14

Thus, the second sentence is true when x is 14, and false otherwise.

These sentences always have the same truth values. The first one is easier to work with.

For all real numbers x, $x = 14$ is equivalent to $\frac{x}{2} = 7$.

For all real numbers x, $x = 14$ if and only if $\frac{x}{2} = 7$.

For all real numbers x, $x = 14$ iff $\frac{x}{2} = 7$.

For all real numbers x, $x = 14 \iff \frac{x}{2} = 7$.

12. (a) false; line 4

(b) false; line 3

(c) true; line 3

(d) false; line 4

13. Both subsentences must be true.

14. At least one of the subsentences is false.

15. At least one of the subsentences is true.

16. Both subsentences must be false.

17. If $S1$ is true, then the 'or' sentence is true. We don't need to know the truth value of $S2$.

18. If $S1$ is false, then the 'and' sentence is false. We don't need to know the truth value of $S2$.

END-OF-SECTION EXERCISES:

19. EXP

20. SEN; ST/SF

21. SEN; ST/SF

22. SEN; T (both subsentences are true)

23. SEN; T (both subsentences are false)

24. SEN; always T (both subsentences *always* have the same truth values)

25. SEN; T (No matter what real number is substituted for x, both subsentences always have the same truth values.)

26. SEN; T (both subsentences are true)

27. SEN; T (an 'or' sentence is true when at least one of the subsentences is true; in this case, both subsentences are true)

28. SEN; F (an 'and' sentence is false when at least one of the subsentences is false)

29. SEN; T

30. SEN; T